Bianchi type IX world with torsion and shear

L. C. Garcia de Andrade[1] and G. A. Monerat[2]

Abstract

Oscillating phases in Bianchi type IX anisotropic universes are obtained as linearized solutions of Einstein-Cartan theory of gravity.Instead of making use of Weyssenhoff spinning fluids we addopt axionic torsion matter where the torsion tensor is totally skew-symmetric as in Kalb-Rammond type theories.Spin-torsion,metric and matter density oscillate at different phases. An upper bound oscillatory limit for torsion is obtained from the model. Metric coefficients correspond to two harmonic oscillators equations,the first describes a harmonic simple oscillator and the second a forced harmonic oscillator.

Recently Harko et al [1] and H.Q.Lu and K.S.Cheng [2] have investigated in detail the Bianchi types I and V cosmological models in the realm of Einstein-Cartan theory of gravity [3]. In this letter a linearized solution of Einstein-Cartan (EC) theory representing a Bianchi type IX [4] cosmological model is presented.The solution possess the interesting feature of being an oscillating solution where the spin-torsion and matter densities oscillate at distinct phasess as well as the metric coefficients. Oscillating universes are important in the reheating phases of the Universe just after inflation. In fact this work in a certain sense is a continuation of our previous paper [4] on inflationary phases as a de Sitter solution of EC gravity.Anisotropic solutions could also lead to topological defects in EC gravity.The physical

[1]garcia@dft.if.uerj.br - Departamento de Fisica Teorica-Instituto de Física -UERJ, Rua São Francisco Xavier 524, Maracanã,R.J Cep: 20550 - Rio de Janeiro, Brasil.

[2]monerat@if.uff.br - Universidade Federal Fluminense, Instituto de Física, Av., Litrônea s/n - Boa Viagem, Cep:24210-340, Niterói ,R.J, Brasil.

motivation of using anisotropic models is according to Harko et al [1] be due to the fact that the spin is polarized. Since we only deal with linearized solution the structure of singularities on these models is out the scope of this work.Nevertheless is important to mention that the vacuum Bianchi type IX exact solution of Einstein field equation of General Relativity is already non-singular. Let us now consider the geometry of Bianchi type IX cosmological model is given by [5]

$$ds^2 = 2\omega^1 dt + g_{11}(\omega)^2 + e^{2\beta}(\omega^2 + \omega^3) \tag{1}$$

where Taub space corresponds to $g_{11} > 0$ and the NUT space corresponds to $g_{11} < 0$.The coordinate t can be parametrized so that $g_{01} = 1$ and in the basis

$$\sigma_1 = \omega^1 \tag{2}$$

and $\sigma^a = e^{\beta}\omega^a$ where $a = 1, 2$ and $d\sigma^2 = a_S \omega^{S^2}$ is the hypersurface metric.Making $g = g_{11}$ the we are left with a off-diagonal metric.Making use of an appropriate frame of differential forms the following components of the Riemannian Einstein tensor is

$$G_{00} = R_{00} \tag{3}$$

and

$$G_{11} = -\frac{1}{2}R \tag{4}$$

where R is the Ricci scalar and $G_{22} = G_{33}$.Component G_{01} is also distinct from zero.Taking the axionic torsion energy-stress tensor as

$$T^{Ax}{}^{l}_{i} = 3S_{ijk}S^{ljk} - \frac{1}{2}\delta^l_i S^2 \tag{5}$$

where $S^2 = S_{ijk}S^{ijk}$ is the the spin-torsion polarized density from the non-symmetric connection and $i, j, k = 0, 1, 2, 3$.Here we assume that the only nonvanishing component of the torsion tensor isS_{012}.The matter stress-energy tensor is given by

$$T_{ij} = \rho u_i u_j \tag{6}$$

which corresponds to the dust pressureless axionic fluid where the four velocity is given by

$$u^i = \delta^i_0 + \delta^i_1 \tag{7}$$

With these tools in hands one may compute the Einstein-Cartan equations as

$$C(t) + \frac{1}{2}e^{-4\beta} = \rho + S^2 \tag{8}$$

and

$$g^2C(t) - \frac{1}{4}g\ddot{g} - \frac{1}{2}ge^{-2\beta} + \frac{1}{4}g^2e^{-4} - \frac{1}{4}\dot{g}\dot{\beta} = \rho(1+g) + S^2 \tag{9}$$

where $C(t) = -2(\ddot{\beta} + \dot{\beta}^2)$.The remaining EC equations are

$$-g\ddot{\beta} + \frac{1}{2}e^{-2\beta} - \frac{1}{4}e^{-4\beta} - \frac{1}{4}\ddot{g} = (1 - \frac{g}{2})S^2 \tag{10}$$

and

$$-g\ddot{\beta} + \frac{3}{4}ge^{-4\beta} + gC - \frac{1}{2}e^{-2\beta} = \rho(1+g) - \frac{S^2}{2} \tag{11}$$

These equations are clearly very complicated to be solved without computers.Since for our physical purposes it is enough to deal with the linear approximation of the EC equations one may reduce the above EC equations to

$$-2\ddot{\beta} + \frac{1}{2}e^{-4\beta} = \rho + S^2 \tag{12}$$

and

$$-\frac{1}{2}g = \rho(1+g) + \frac{1}{2}S^2 \tag{13}$$

and the remaining two equations are

$$\frac{1}{2}e^{-2\beta} - \frac{1}{4}g + \frac{1}{4}\ddot{g} = S^2 \tag{14}$$

and

$$-\frac{3}{4}g = \rho(1+g) - \frac{1}{2}S^2 \tag{15}$$

After some algebra we reduce these last four field equations to

$$\ddot{\beta} + \beta = -\frac{5}{8}g \tag{16}$$

and

$$\ddot{g} + 4g = 0 \tag{17}$$

The last equation is a very simple differential equation describing an harmonic oscillator with constant frequency which yields the solution

$$g = cos2t \tag{18}$$

which gives the first hint of the oscillatory behaviour of this space-time.If the space will be of Taub type $g = g_{11} > 0$ or of NUT type $g = g_{11} < 0$ will depend upon the time argument.Substitution of this solution on the remaining equations of the system yields

$$\rho = \frac{\frac{1}{8}cos2t}{1 + cos2t} \tag{19}$$

we end up also with a slightly more complicated equation for the $\beta(t)$ metric function such as

$$\ddot{\beta} + {\omega_0}^2\beta = f(t) \tag{20}$$

this represents an equation for the forced harmonic oscillator where the square of the frequency is

$${\omega_0}^2 = \frac{3}{2} \tag{21}$$

and the force $f(t)$ reads

$$f(t) = \frac{5}{8}cos2t \tag{22}$$

The equation (20) solves to

$$\beta = \frac{1}{4}cos2t \tag{23}$$

.From expression (19) one obtains immeadiatly a upper bound limit for torsion $S^2 < \frac{1}{8}cos2t$ by simply imposing the positivity of the matter density $\rho > 0$.Here we have consider approximation by dropping terms like g^2.This shows that the spin-torsion density is also oscillatory.Substitution of (23) into equation (16) allow us to determine $S^2 = \frac{1+\frac{19}{8}cos2t}{1+cos2t}$. An exact solution of this model can be obtained in the near future along with a detailed investigation of the singularity problem in Riemann-Cartan space for this Bianchi Type IX model.This in fact is not the first time that oscillations in the Universe appears in non-Riemannian cosmology.In 1983 Nurgaliev and Ponomariev [6] have investigated the evolution of the early universe in EC gravity where the problems of gravitational instabilities have been considered, wherethe solution seems to be stable to small homogeneous perturbations and an oscillatory phase is found before inflation.Two graphics are displayed on this text.The first two represent the behaviour of the metric factors g and β which allows us to say that the it behaves oscillatory while its amplitude is distinct in different directions. This behaviour is similar to the one obtained by Novikov and Zeldovich [6] who investigated the evolutionary stages of the

Bianchi type IX universe in spacetimes without torsion. The behaviour for the square of torsion is obtained in graphic 3. More recently we show that teleparallel spaces can induce a cosmological model where oscillation on the cosmic scale factor are present. In this case the frequency of oscillations are damped by the presence of torsion. In these two cases contrary to what happens here both metrics are isotropic. One another important feature of the present work is that our metric presents shear besides rotation. A more detailed investigation of oscillatory cosmological models in Riemann-Cartan space-times and its implications to inflation may appear elsewhere.

Acknowledgements

We would like to express our gratitude to CNP_q.(Brazil) from financial support. I also thank Universidade do Estado do Rio de Janeiro for partial financial support. Thanks are also due to Prof.H.P.de Oliveira for helpful discussions on the subject of this paper and to Prof. Yuri N. Obukhov for point it out an error in the first draft of this paper.

Figure 1: metric factor-g

Figure 2: metric factor-β

Figure 3: Torsion decay

References

[1] T.Harko,M.K.Mak,H.Q.Lu and K.S.Cheng,Il Nuovo Cimento 114 B,N.12,(1999),1389.

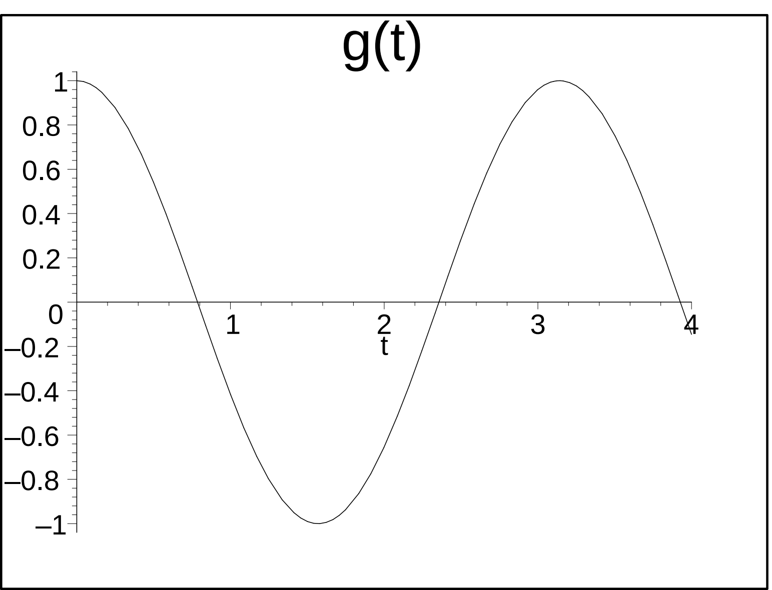
[2] H.Q.Lu and K.S.Cheng,Class. and Quantum Grav.12(1995),2755.

[3] V. de Sabbata and C.Sivaram,Spin and Torsion in Gravitation,(1994),world Scientific.

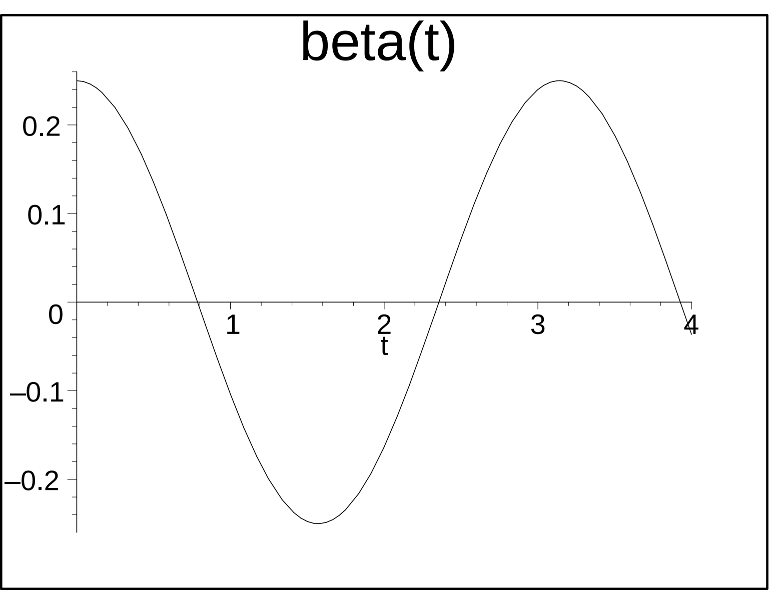
[4] L.C.Garcia de Andrade,Phys.Lett.B,468,28.

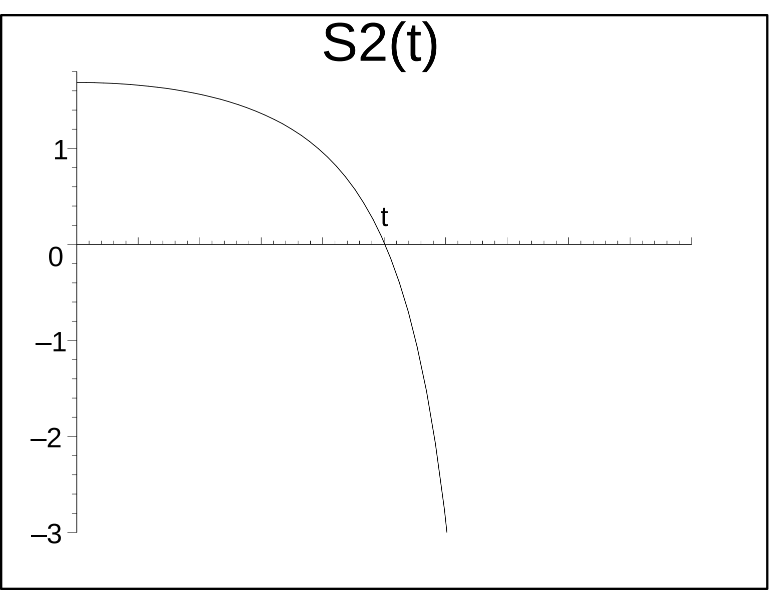
[5] M.P.Ryan Jr. and L.C.Shepley,Homogeneous Relativistic Cosmologies,(1975),Princeton University Press.

[6] I. D. Novikov and Ya. B. Zeldovich,Physical Processes near Cosmological Singularities,Annual Review of Astronomy and Astrophysics,volume II (1973).



g(t)
1
0.8
0.6
0.4
0.2
0
–0.2
–0.4
–0.6
–0.8
–1
1
2
3
4
t


beta(t)
0.2
0.1
0
–0.1
–0.2
1
2
3
4
t


S2(t)
1
0
t
−1
−2
−3